International Journal of Theoretical Physics, Vol. 29, No. 4, 1990

Supercharacters and Superdimensions of Irreducible Representations of $B(O/s)$ Orthosymplectic Simple Lie Superalgebras

I. Tsohantjis[1] and J. F. Cornwell[1]

Received March 7, 1989

The validity of different formulas in the literature for the supercharacters and superdimensions of the irreducible representations of the $B(O/s)$ orthosymplectic simple Lie superalgebras is critically examined.

1. INTRODUCTION

The simple Lie superalgebras have been widely studied in the mathematics and mathematical physics literature not only for their intrinsic interest, but also for their wide range of application in theoretical physics, of which the papers of D'Auria and Fré (1982), Delbourgo and Jarvis (1983), Leites and Serganov (1984), Morel *et al.* (1986), Van Nieuwenhuizen (1986), Casalbouni *et al.* (1987), Siegel (1987), Siegel and Zwiebach (1987), and Beckers and Cornwell (1989*a*,*b*) may be mentioned as a small but fairly representative collection.

The purpose of this note is to clarify the situation concerning the supercharacters and superdimensions of the irreducible representations of the simple Lie superalgebras $B(O/s)$ (for $s \geq 1$). The original study of characters, supercharacters, dimensions, and superdimensions for the irreducible representations of all the simple Lie superalgebras was made by Kac in a set of very important and stimulating papers. His results were reported first in a brief note (Kac 1977*a*), were quoted again without amplification in a short review (1977*c*), and were then extended with more details of the derivations in a more elaborate exposition (Kac, 1978). Unfortunately, the conclusions for the special case of the Lie superalgebras

[1]Department of Physics and Astronomy, University of St. Andrews, St. Andrews, Fife, KY16 8RT, Scotland.

0020-7748/90/0400-0351$06.00/0

$B(O/s)$ (for $s \geq 1$) are rather confusing in several respects. First, although the expressions for the supercharacters and the superdimensions are correctly quoted in the original note [Kac (1977*a*), Corollary 3 and the statement immediately preceding it], in the expanded version (Kac, 1978) it is erroneously stated (in a footnote on p. 619) that this original expression for the supercharacters contains a misprint, an expression being given [in equation (2.5)] that differs from the original by a certain power of 2 for $s \geq 2$. Moreover, a formula [equation (2.7)] involved in the explicit derivation of this second expression for the supercharacters in this expanded treatment itself contains an incorrect factor. Second, this expanded version gives [in equation (2.6)] an incorrect expression for the superdimension, a factor of a power of 2 being missing (for $s \geq 2$). Third, certain equalities that were employed in the expanded version to produce a modified version of the superdimension formula are themselves incorrect. Nevertheless, as will be shown in this note, this modified version of the superdimension [equation (11) below] is actually correct, for, by a remarkable coincidence, the insertion of the corrected versions of these equalities in the corrected version of equation (2.6) of Kac (1978) produces a factor that exactly cancels the power of 2 mentioned above. Finally, the result for the superdimension mentioned in Kac (1977*c*) is different from all the expressions just mentioned.

It is important to note that all the expressions quoted by Kac (1977*a*, 1978) for the "non-super" characters and dimensions of irreducible representations are correct, as is the statement that the superdimension is zero for every "typical" irreducible representation of all the simple Lie superalgebras other than $B(O/s)$ (for $s \geq 1$).

The plan of this paper is as follows. In Section 2 the necessary properties of the Lie superalgebras $B(O/s)$ are briefly stated. The derivation of the formulas for the supercharacters and superdimensions is then described in Section 3. The general line of argument is the same as in the papers of Kac (1977*a*, 1978) [which is itself an adaptation of that introduced by Weyl (1925, 1926*a*,*b*) for characters and dimensions of the irreducible representations of the semisimple Lie algebras], with the emphasis here being on the points where the additional factors mentioned above appear.

For further information on the simple Lie superalgebras see Kac (1977*b*,*c*), Scheunert (1979), and Cornwell (1989).

2. SOME PROPERTIES OF THE SIMPLE LIE SUPERALGEBRA $B(O/s)$ (FOR $s \geq 1$)

A realization of $B(O/s)$ (for $s \geq 1$) is provided by the orthosymplectic algebra $osp(1/2s; \mathbb{C})$ considered as a complex Lie superalgebra. This may be defined as the set of $(2s+1)\times(2s+1)$ matrices $\mathbf{M}$ with complex entries and with the partitioning

$$\mathbf{M}=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}$$

where **A**, **B**, **C**, and **D** are submatrices with dimensions 1×1, $1\times2s$, $2s\times1$, and $2s\times2s$, respectively, that satisfy the condition

$$\mathbf{M}^{\mathrm{st}}\mathbf{K}+(-1)^{\deg\mathbf{M}}\mathbf{K}\mathbf{M}=\mathbf{0}$$

Here deg(**M**) denotes the degree of **M**, $\mathbf{M}^{\mathrm{st}}$ denotes the supertranspose of **M**, and

$$\mathbf{K}=\begin{bmatrix}\mathbf{1}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{J}\end{bmatrix}$$

where **J** is the $2s\times2s$ matrix

$$\mathbf{J}=\begin{bmatrix}\mathbf{0} & \mathbf{1}_s\\ -\mathbf{1}_s & \mathbf{0}\end{bmatrix}$$

and $\mathbf{1}_p$ is the $p\times p$ unit matrix. The even elements of $B(O/s)$ are therefore of the form

$$\mathbf{M}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{D}\end{bmatrix}$$

where the matrices **D** must satisfy the constraint $\tilde{\mathbf{D}}\mathbf{J}+\mathbf{J}\mathbf{D}=\mathbf{0}$. Similarly, the odd elements of $B(O/s)$ are of the form

$$\mathbf{M}=\begin{bmatrix}\mathbf{0} & \mathbf{B}\\ \mathbf{C} & \mathbf{0}\end{bmatrix}$$

and so satisfy the constraint $\tilde{\mathbf{B}}\mathbf{G}=\mathbf{J}\mathbf{C}$. The dimensions of the even and odd parts of $B(O/s)$ are $s(2s+1)$ and $2s$, respectively. The even subalgebra of $B(O/s)$ is isomorphic to the simple complex Lie algebra C_s, and the rank of $B(O/s)$ is s.

A convenient basis for the Cartan subalgebra $\mathscr{H}$ of $B(O/s)$ is provided by the $(2s+1)\times(2s+1)$ matrices $\mathbf{h}_j$, where $(\mathbf{h}_j)_{pq}=\delta_{p,j+1}\delta_{q,j+1}-\delta_{p,j+s+1}\delta_{q,j+s+1}$ (for $p, q=1,\dots,2s+1$). Using these, a set of s linear functionals $\varepsilon_1,\varepsilon_2,\dots,\varepsilon_s$ may be defined on the Cartan subalgebra $\mathscr{H}$ by the prescription $\varepsilon_p(h_j)=\delta_{pj}$ (for $j, p=1,2,\dots,s$). In terms of these, the set Δ_0^+ of *positive even* roots of $B(O/s)$ consists of the s^2 linear functionals

$$\varepsilon_p-\varepsilon_q \qquad (\text{for } p, q=1,2,\dots,s, \text{ with } p<q)$$

and

$$\varepsilon_p+\varepsilon_q \qquad (\text{for } p, q=1,2,\dots,s, \text{ with } p\le q)$$

whereas the set Δ_1^+ of *positive odd* roots of $B(O/s)$ just consists of the s linear functionals

$$\varepsilon_p \qquad (\text{for } p=1,2,\ldots,s)$$

For all of these roots the corresponding root subspace is one-dimensional, and to each one there is an associated negative root. Moreover, defining $\bar{\Delta}_0^+$ as the subset of roots α of Δ_0^+ that are such that $\frac{1}{2}\alpha$ is not an odd root, and $\bar{\Delta}_1^+$ as the subset of roots α of Δ_1^+ that are such that 2α is not an even root, it is clear that $\bar{\Delta}_0^+$ is empty if $s=1$, but for $s\geq 2$

$$\bar{\Delta}_0^+=\{\varepsilon_p\pm\varepsilon_q \,|\, \text{for } p, q=1,2,\ldots,s, \text{ with } p<q\} \tag{1}$$

and its complement $\Delta_0^+-\bar{\Delta}_0^+$ in Δ_0^+ is given (for $s\geq 1$) by

$$\Delta_0^+-\bar{\Delta}_0^+=(2\varepsilon_p \,|\, \text{for } p=1,2,\ldots,s\}$$

whereas $\bar{\Delta}_1^+$ is empty for all $s\geq 1$. With ρ and $\bar{\rho}_0$ defined by

$$\rho=\tfrac{1}{2}\sum_{\alpha\in\Delta_0^+}\alpha-\tfrac{1}{2}\sum_{\alpha\in\Delta_1^+}\alpha$$

and

$$\bar{\rho}_0=\tfrac{1}{2}\sum_{\alpha\in\bar{\Delta}_0^+}\alpha$$

it follows that for $B(O/s)$ with $s\geq 2$

$$\rho=\tfrac{1}{2}\{(2s+1)\varepsilon_1+(2s-1)\varepsilon_2+\cdots+5\varepsilon_{s-2}+3\varepsilon_{s-1}+\varepsilon_s\} \tag{2}$$

and

$$\bar{\rho}_0=(s-1)\varepsilon_1+(s-2)\varepsilon_2+\cdots+2\varepsilon_{s-2}+\varepsilon_{s-1} \tag{3}$$

where, it should be noted, the linear functional ε_s does not appear in $\bar{\rho}_0$. For $s=1$, $\rho=3\varepsilon_1/2$ and $\bar{\rho}_0=0$.

The Killing form $B(\cdot,\cdot)$ of $B(O/s)$ is given by

$$B(\mathbf{M},\mathbf{N})=-(2s+1)\,\mathrm{str}(\mathbf{MN})$$

for all $\mathbf{M}$ and $\mathbf{N}$ of $B(O/s)$. For any linear functional α defined on the Cartan subalgebra $\mathscr{H}$ an element h_α can be defined by

$$B(h_\alpha,h)=\alpha(h) \qquad \text{for all} \quad h\in\mathscr{H}$$

Thus,

$$h_{\varepsilon_p}=\{1/2(2s+1)\}h_p \qquad \text{for} \quad p=1,2,\ldots,s$$

Defining $\langle\alpha,\beta\rangle$ by $\langle\alpha,\beta\rangle=B(h_\alpha,h_\beta)$ for any pair of linear functionals α and β defined on $\mathscr{H}$, it follows that

$$\langle\varepsilon_p,\varepsilon_q\rangle=\{1/2(2s+1)\}\delta_{pq} \qquad \text{for} \quad p,q=1,2,\ldots,s \tag{4}$$

The Weyl group $\mathcal{W}$ of $B(O/s)$ is isomorphic to that of its even subalgebra C_s, and so has dimension dim $\mathcal{W}$ given by dim $\mathcal{W} = s!2^s$. It has the semidirect product structure $\mathcal{W} = \mathcal{T}Ⓢ\mathcal{W}_0$, which is most easily described in terms of the linear functionals ε_p. Let

$$\kappa_1\varepsilon_1(h) + \kappa_2\varepsilon_2(h) + \cdots + \kappa_s\varepsilon_s(h)$$

be any linear functional defined on the Cartan subalgebra of $B(O/s)$, so that $\{\kappa_1, \kappa_2, \ldots, \kappa_s\}$ is a set of complex numbers. Then the invariant subgroup $\mathcal{T}$ may be defined to consist of the 2^s operations that change the sign of members of the set $\{\kappa_1, \kappa_2, \ldots, \kappa_s\}$, while $\mathcal{W}_0$ may be defined as the set of $s!$ operations that permute the members of the set $\{\kappa_1, \kappa_2, \ldots, \kappa_s\}$. It is easily shown that $\mathcal{T}$ is generated by reflections with respect to the roots of $\Delta_0^+ - \bar{\Delta}_0^+$, while $\mathcal{W}_0$ is generated by a subset of the reflections with respect to the roots of $\bar{\Delta}_0^+$ (and consists only of the identity if $s = 1$). Defining $\mathscr{E}(S)$ and $\mathscr{E}'(S)$ by $\mathscr{E}(S) = (-1)^R$ and $\mathscr{E}'(S) = (-1)^{R'}$, where R is the number of reflections that appear when S is written as a product of Weyl reflections and R' is the number of Weyl reflections with respect to the roots of $\bar{\Delta}_0^+$ that appear in this expansion, it can be shown that $\mathscr{E}'(S_0T) = \mathscr{E}(S_0)$ for all $S_0 \in \mathcal{W}_0$ and all $T \in \mathcal{T}$.

Also needed is the subgroup $\mathcal{T}'$ of $\mathcal{T}$ that is defined to consist of all the operations of $\mathcal{T}$ that leave the sign of the last member κ_s of the set $\{\kappa_1, \kappa_2, \ldots, \kappa_s\}$ unchanged, and the subset $\mathcal{T}''$ of $\mathcal{T}$ that consists of all the operations of $\mathcal{T}$ that change the sign of κ_s. It is obvious that $\mathcal{T}$ and $\mathcal{T}'$ both contain 2^{s-1} operations, and that $\mathcal{T} = \mathcal{T}' \cup \mathcal{T}''$. Indeed, if T_s is the operation of $\mathcal{T}$ that changes the sign of κ_s but leaves unchanged the signs of all the other κ_p, then the correspondence

$$T'' = T_sT' \tag{5}$$

provides a one-to-one mapping between $\mathcal{T}''$ and $\mathcal{T}'$. The corresponding subsets $\mathcal{W}'$ and $\mathcal{W}''$ of $\mathcal{W}$ may then be defined by

$$\mathcal{W}' = \{S_0T' \mid \text{for all } S_0 \in \mathcal{W}_0 \text{ and all } T' \in \mathcal{T}'\}$$

and

$$\mathcal{W}'' = \{S_0T'' \mid \text{for all } S_0 \in \mathcal{W}_0 \text{ and all } T'' \in \mathcal{T}''\}$$

Clearly, both $\mathcal{W}'$ and $\mathcal{W}''$ contain $s!2^{s-1}$ operations and $\mathcal{W} = \mathcal{W}' \cup \mathcal{W}''$.

A further ingredient that is needed is the quantity L' defined by

$$L' = \prod_{\alpha \in \Delta_0^+} (e^{\alpha/2} - e^{-\alpha/2}) \Big/ \prod_{\alpha \in \Delta_1^+} (e^{\alpha/2} - e^{-\alpha/2})$$

Finally, if α and β are any two linear functionals defined on $\mathcal{H}$, and if t is a real variable, the real-valued functional $f_\beta(e^\alpha)$ is defined by

$$f_\beta(e^\alpha) = e^{t\langle\alpha,\beta\rangle}$$

3. THE SUPERCHARACTERS AND SUPERDIMENSIONS

Consider an irreducible representation of $B(O/s)$ (for $s \geq 1$) with highest weight Λ and carrier space V, the even and odd parts of which are V_0 and V_1, respectively. The convention will be adopted that these subspaces are chosen so that the weight subspace corresponding to Λ is a member of V_0. Let d_0 and d_1 be the dimensions of V_0 and V_1. Then the superdimension of this representation sdim V is defined by

$$\text{sdim } V = \{d_0 - d_1\}$$

Let $D(\Lambda)$ be the set of weights of this representation, let the multiplicity of a typical weight λ of $D(\Lambda)$ be denoted by $m(\lambda)$, and let $\omega(\lambda)$ be defined to have the values 1 or -1 depending on whether the weight space corresponding to λ is a member of V_0 or V_1. The supercharacter sch V of this representation is then defined by

$$\text{sch } V = \sum_{\lambda \in D(\Lambda)} \omega(\lambda) m(\lambda)\, e^{\lambda}$$

As noted by Kac (1977*a*, 1978), the relationship between the supercharacter and the superdimension is given by

$$\text{sdim } V = \lim_{t \to 0} f_{\bar{\rho}_0}(\text{sch } V)$$

The quantity on the right-hand side can be evaluated by using the identity

$$\text{sch } V = (L')^{-1} \sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{S(\Lambda+\rho)}$$

[cf. Kac (1978), equation (2.2)], together with the observation that

$$\begin{aligned} f_{\bar{\rho}_0}\left(\sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{S(\Lambda+\rho)} \right) &= \sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{t\langle S(\Lambda+\rho), \bar{\rho}_0 \rangle} \\ &= \sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{t\langle \Lambda+\rho, S(\bar{\rho}_0) \rangle} \\ &= f_{\Lambda+\rho}\left(\sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{S(\bar{\rho}_0)} \right) \end{aligned}$$

However, as will be shown shortly, for $s \geq 2$

$$\sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{S(\bar{\rho}_0)} = 2 \prod_{\alpha \in \bar{\Delta}_0^+} \{e^{\alpha/2} - e^{-\alpha/2}\} \tag{6}$$

while it is obvious that for $s = 1$

$$\sum_{S \in \mathscr{W}} \mathscr{E}'(S)\, e^{S(\bar{\rho}_0)} = 2$$

[as in this case $\bar{\rho}_0 = 0$, dim $\mathscr{W} = 2$, and $\mathscr{E}'(S) = 1$ for both elements S of $\mathscr{W}$, as $\bar{\Delta}_0^+$ is empty]. Thus, for $s \geq 2$

$$f_{\bar{\rho}_0}(\text{sch } V) = \left[2 \prod_{\alpha \in \bar{\Delta}_0^+} (e^{t\langle\alpha,\Lambda+\rho\rangle/2} - e^{-t\langle\alpha,\Lambda+\rho\rangle/2}) \prod_{\alpha \in \Delta_1^+} (e^{t\langle\alpha,\bar{\rho}_0\rangle/2} - e^{-t\langle\alpha,\bar{\rho}_0\rangle/2})\right] \times \left[\prod_{\alpha \in \Delta_0^+} (e^{t\langle\alpha,\bar{\rho}_0\rangle/2} - e^{-t\langle\alpha,\bar{\rho}_0\rangle/2}\right]^{-1}$$

and for $s = 1$

$$f_{\bar{\rho}_0}(\text{sch } V) = \left[2 \prod_{\alpha \in \Delta_1^+} (e^{t\langle\alpha,\bar{\rho}_0\rangle/2} - e^{-t\langle\alpha,\bar{\rho}_0\rangle/2}\right] \times \left[\prod_{\alpha \in \Delta_0^+} (e^{t\langle\alpha,\bar{\rho}_0\rangle/2} - e^{-t\langle\alpha,\bar{\rho}_0\rangle/2})\right]^{-1}$$

As $2\alpha \in \Delta_0^+ - \bar{\Delta}_0^+$ if and only if $\alpha \in \Delta_1^+$, it follows that for $s \geq 2$

$$\text{sdim } V = \left(2 \prod_{\alpha \in \bar{\Delta}_0^+} \langle\Lambda+\rho,\alpha\rangle\right) \Bigg/ \left(2^{N_1^+} \prod_{\alpha \in \bar{\Delta}_0^+} \langle\bar{\rho}_0, \alpha\rangle\right)$$

where N_1^+ is the number of roots in Δ_1^+, so that, as $N_1^+ = s$,

$$\text{sdim } V = 2^{1-s} \prod_{\alpha \in \bar{\Delta}_0^+} \{\langle\Lambda+\rho, \alpha\rangle / \langle\bar{\rho}_0, \alpha\rangle\} \qquad \text{for} \quad s \geq 2 \tag{7}$$

Similarly,

$$\text{sdim } V = 1 \qquad \text{for} \quad s = 1$$

The expression for sdim V given in equation (2.6) of Kac (1978) has the same form as (7), but the factor 2^{1-s} is missing. This discrepancy can be traced back to (6), for which Kac (1978) quotes [in equation (2.7)] a similar identity, but with the factor 2 on the right-hand side replaced by dim $\mathscr{T}$, the order of the invariant subgroup $\mathscr{T}$ of the Weyl group $\mathscr{W}$, which for $B(O/s)$ is given by dim $\mathscr{T} = 2^s$.

The correctness of (6) will now be demonstrated, the argument being concentrated on the numerical factor appearing on the right-hand side. It can be shown without difficulty that

$$\sum_{S \in \mathscr{W}'} \mathscr{E}'(S)\, e^{S(\bar{\rho}_0)} = \prod_{\alpha \in \bar{\Delta}_0^+} \{e^{\alpha/2} - e^{-\alpha/2}\} \tag{8}$$

the easiest way of checking the factor being to note the coefficient of the exponent with the highest exponent

$$\bar{\rho}_0 = \tfrac{1}{2} \sum_{\alpha \in \bar{\Delta}_0^+} \alpha$$

on the right-hand side is 1 (as there is only one way of getting this term), and this is the coefficient of the corresponding term on the left-hand side [as $S(\bar{\rho}_0) \neq \bar{\rho}_0$ for all S of $\mathscr{W}'$ other than the identity]. Moreover, as $\mathscr{E}'(T'') = \mathscr{E}'(T')$ in the mapping (5), and as $T_s(\bar{\rho}_0) = \bar{\rho}_0$, it is also true that

$$\sum_{S \in \mathscr{W}''} \mathscr{E}'(S)\, e^{S(\bar{\rho}_0)} = \prod_{\alpha \in \bar{\Delta}_0^+} \{e^{\alpha/2} - e^{-\alpha/2}\} \tag{9}$$

The identity (7) then follows from (8) and (9) as $\mathscr{W} = \mathscr{W}' \cup \mathscr{W}''$.

Our last point concerns the relationship of the quantities $\langle \rho, \alpha \rangle$ and $\langle \bar{\rho}_0, \alpha \rangle$ for $\alpha \in \bar{\Delta}_0^+$. The statement at the top of p. 620 of Kac (1978) that $\langle \rho, \alpha \rangle = \langle \bar{\rho}_0, \alpha \rangle$ for *all* $\alpha \in \bar{\Delta}_0^+$ is not correct. However, it is true that

$$\langle \rho, \varepsilon_p - \varepsilon_q \rangle / \langle \bar{\rho}_0, \varepsilon_p - \varepsilon_q \rangle = 1 \qquad \text{for} \quad p, q = 1, 2, \ldots, s, \quad \text{with } p < q$$

[by (2)-(4)], whereas

$$\prod_{q=p+1}^{s} \{\langle \rho, \varepsilon_p + \varepsilon_q \rangle / \langle \bar{\rho}_0, \varepsilon_p + \varepsilon_q \rangle\} = 2 \qquad \text{for} \quad p = 1, 2, \ldots, s-1$$

so, from (1),

$$\prod_{\alpha \in \bar{\Delta}_0^+} \{\langle \rho, \alpha \rangle / \langle \bar{\rho}_0, \alpha \rangle\} = 2^{s-1} \tag{10}$$

Thus, (7) can be rewritten as

$$\text{sdim } V = \prod_{\alpha \in \bar{\Delta}_0^+} \{\langle \Lambda + \rho, \alpha \rangle / \langle \rho, \alpha \rangle\} \qquad \text{for} \quad s \geq 2 \tag{11}$$

the factor of 2^{s-1} from (10) canceling the factor of 2^{1-s} in (7).

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